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Shearing the vacuum—quantum friction

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Abstract. We consider two perfectly smooth featureless surfaces at $T = 0$, defined only by their respective dielectric functions, separated by a finite distance, and ask the question of whether they can experience any friction when sheared parallel to their interface. We find large frictional effects comparable to everyday frictional forces provided that the materials have resistivities of the order of $1 \text{ m } \Omega$ and that the surfaces are in close proximity. The friction depends solely on the reflection coefficients of the surfaces for electromagnetic waves and its detailed behaviour with shear velocity and separation is dictated by the dispersion of the reflectivity with frequency.

1. Introduction

Friction is a problem of great practical importance, usually associated with complex systems that are difficult to characterize. There has been much discussion of the mechanisms lying behind friction [1, 2] but despite the many profound insights no all-embracing theory has been formulated to date and it is not the objective of this paper to propose one. Instead our aim is to focus on the simplest possible system, bereft of every complexity, and ask how friction arises. For simplicity we restrict ourselves to $T = 0$.

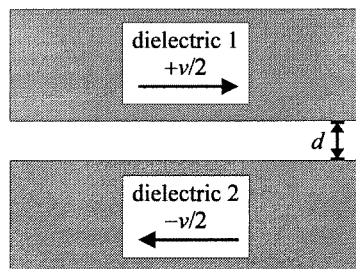


Figure 1. Perfectly smooth dielectric surfaces shear against one another with relative velocity v . Is there a frictional force?

We consider two perfectly smooth featureless surfaces, parallel but not in contact, defined only by their electromagnetic reflection coefficients. We now shear the surfaces with relative velocity v and calculate the friction; see figure 1. Note the absence any roughness, a quality normally associated with friction. How is one surface ‘aware’ of the other’s motion? We can detect relative motion of a dielectric surface: reflect an electromagnetic wave from the surface and the reflection coefficient will show an asymmetry along the direction of motion; see figure 2.

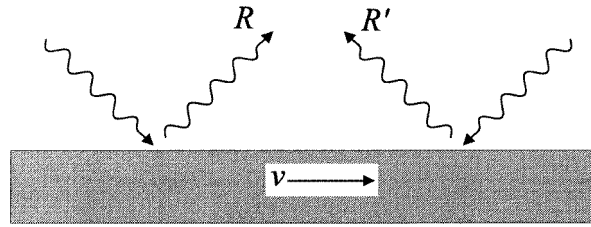


Figure 2. We can detect the motion of a smooth dielectric surface by measuring its reflection coefficient with and against the motion as shown in the figure. In the reference frame of the dielectric the two incident waves are Doppler shifted in opposite senses and therefore reflect differently from the surface.

The two incident waves experience opposite Doppler shifts in the reference frame of the dielectric and, assuming dispersion with frequency, the two waves experience different reflection coefficients. If the surfaces are hot they will naturally emit radiation, but even a cold surface will be surrounded by a radiation field due to zero-point quantum fluctuations. This exchange of virtual photons is the frictional mechanism that we study in this paper. We shall show, contrary to a natural suspicion that such an effect should be extremely small, that the effect is large: comparable to other contributions to friction.

Our point of view is that friction is the exchange of momentum between two surfaces and therefore we must be able to describe this process as an exchange of particles since all forces are ultimately mediated by a particle. In the case of friction with wear, atoms are exchanged between the surfaces either singly or in larger fragments. This is the classic frictional mechanism to which we can all relate: the grating of two rough surfaces in intimate contact. But it is only one mechanism, and it is understood that friction can occur even when there is negligible wear. Other means of momentum exchange are possible.

Another particle that can be exchanged is the electron. This need not imply charging of the surfaces provided that equal numbers flow in opposite directions. Obviously metals, where the electrons are freely moving, are the prime candidates. Since electron density decreases exponentially outside a surface this mechanism will be short ranged, limited to a scale dictated by the work function. We shall give brief consideration to this mechanism and show that it creates forces comparable to our electromagnetic mechanism.

Finally there is the photon. This may mediate the electrostatic forces operating between charge distributions on opposite surfaces, or it may play a more subtle role involving the zero-point fluctuations. Forces involving photons will be long ranged because there is no work function preventing the photon's escape from a surface. It is with the photon that we shall concern ourselves here. We do not seek a universal explanation of friction. Our aim is far more modest: only to demonstrate the simplest possible model that exhibits friction by exchange of photons.

Frictional forces on moving charges outside dielectrics have been studied for many years in the context of electron microscopy and nuclear radiation [3–9] and are well understood. The system with which we are concerned is different: the two surfaces that we shear against one another are assumed to be locally electrically neutral. Quantum mechanically the dielectrics will experience internal charge fluctuations, and images of these charges in the other dielectric will create forces. We are already familiar with the force normal to the interface: the Van der Waals force, but there is in addition a parallel component because the image will lag slightly behind the charge creating it. We find that both forces can be calculated rather simply in terms of the reflection coefficients of the two surfaces.

Furthermore their magnitude is not small provided that the surfaces are in close contact as in a normal friction experiment, and provided that the surfaces have a resistivity of the order of $1 \text{ m } \Omega$. This latter condition relates to the density of electromagnetic states available for dissipating energy. Immediately outside a surface this is proportional to

$$\Im R(\omega) \quad (1)$$

where $R(\omega)$ is the reflection coefficient of the surface. For a purely resistive sample this reduces to

$$\Im \frac{i\sigma/\omega\epsilon_0}{2 + i\sigma/\omega\epsilon_0} \quad (2)$$

where σ is the conductivity. The effect is that of a washboard in which excitations on opposite surfaces with the same wave vector, k , grate against one another with frequency

$$\omega = kv. \quad (3)$$

Given that the longest wave vectors generally correspond to wavelengths no less than an atomic spacing or so, typical velocities of 1 m s^{-1} will result in frequencies in the low GHz range. Adjusting the conductivity, σ , to maximize equation (2),

$$\sigma \approx 1 \text{ m}^{-1} \Omega^{-1}. \quad (4)$$

To exchange momentum between the surfaces we need to create an excitation on each of the surfaces with equal and opposite momentum, not necessarily with equal energy though the total energy must sum to

$$E = \hbar\omega = \hbar kv. \quad (5)$$

Thus energy radiates from the interface in correlated pairs of excitations.

The possibility of radiating energy from dielectrics in motion is not a new one. Perhaps the most celebrated antecedent is the accelerated-mirror concept: a mirror accelerated in the vacuum can be expected to produce electromagnetic radiation through its interaction with vacuum fluctuations [10]. More recently it has been suggested that sonoluminescence can be due to the acceleration of a dielectric fluid [11, 12], though there is still some controversy about whether there is enough acceleration to give the observed effect [13]. All of these phenomena are characterized by the extremely small amounts of radiation predicted and the difficulty of detecting it. Acceleration is a motion that cannot be removed by change of reference frame. A shear motion is another and we shall show that energy is also radiated in this situation. The big difference from the accelerated dielectric is that relatively large amounts of energy can be radiated under conditions of shear.

The possibility of a dissipative component to Van der Waals forces has been considered in earlier work. In some instances the authors have been interested in questions other than frictional forces [14], and in other instances the formulae obtained differ from ours in crucial respects: for example the paper by Teodorovitch [15]. Schaich and Harris [14] argue that Teodorovitch is in error and that is our conclusion too. Levitov [16] presents some calculations of similar quantities, but without giving details, and his conclusions differ in some important aspects from those presented here: in particular his estimate of the forces is very small compared to ours. The work closest to ours is that by Annett and Echenique [17, 18] on the friction experienced by a neutral atom above a surface. Barton has considered the case of partially silvered mirrors sheared against one another, where relativistic effects are vital [19]. Polevoi [20] has also discussed viscosity of the vacuum and his work has some features in common with our ‘poor man’s derivation’ of section 2. Mkrtchian [21] has also discussed the problem but like Levitov gives few details. Liebsch [22] also considered

frictional forces on neutral atoms at surfaces. Høye and Brevik [23, 24] consider the interaction between moving oscillators and treat the case of retardation.

In this paper we give for the first time a simple and general derivation of the frictional forces between dielectrics. We draw attention for the first time to the important fact that if this frictional mechanism is to produce large effects, the relevant electromagnetic density of states must be maximized. This means choosing a resistivity of the order of $1 \text{ m } \Omega$ as discussed above. We have eschewed complex diagrammatic formulations of this simple problem. They obscure the clarity of the situation and are prone to user error when the system is time dependent. Instead we give two derivations. The first given in section 2 is an intuitive one that produces the main results quickly and in some generality. We re-derive the same result in sections 3 and 4 by considering a simple model of a dielectric which dissipates energy through a set of harmonic oscillators, solving that model using a Lagrangian formulation to construct a quantum mechanical equation of motion for the harmonic oscillators. The latter method sheds light on the quantum mechanical processes at work, and highlights the two-photon nature of the frictional process.

By way of comparison we analyse in section 5 the frictional forces generated by exchange of electrons and show that these forces are of similar magnitude to photon-based forces. Finally we discuss the possibility that light may be emitted as part of the frictional process and show that this possibility does not occur within our simple model, unless the dielectrics are sheared at unreasonably large velocities.

2. ‘Poor man’s derivation’ of quantum friction

The final result for quantum friction is a simple one, and leads us to suspect that we could find a simple way of deriving it. It turns out that this is indeed that case, and in the process provides an interesting link with the conventional Van der Waals force between two surfaces.

In the presence of a surface, the polarization of radiation is described as being either of p type where the electric field vector lies in the same plane as the wave vector and surface normal, or s type where the electric field vector lies perpendicular to the plane containing the wave vector and surface normal. In the limit of very large momentum parallel to the surface, which will be important for our calculations, the normal component of the wave vector becomes imaginary describing an exponentially decaying wave. At the same time the p wave becomes almost entirely electrostatic in character, and the s wave almost entirely magnetic. Thus if the surface has magnetic permeability of unity, only the p wave is relevant under these conditions. A more detailed discussion of this limit can be found in the paper by Pendry and Martín-Moreno [8].

Imagine that we have a wave incident on a surface,

$$E_0 \hat{\mathbf{K}}_p^+ \exp(ik_x x + ik_y y + iK_z z) \quad (6)$$

where the polarization is chosen to be p type and we work in the electrostatic limit neglecting the velocity of light,

$$\hat{\mathbf{K}}_p^\pm = \frac{c_0}{\omega} \left[k_x \quad k_y \quad K_z = \pm i \sqrt{k_x^2 + k_y^2} \right]. \quad (7)$$

In this limit the contribution of the s-polarized state is negligible. Reflection from the surface results in a total wavefield of

$$\begin{aligned} \mathbf{E}(\mathbf{r}) = & E_0 \hat{\mathbf{K}}_p^+ \exp(ik_x x + ik_y y + iK_z z) \\ & + E_0 R_{1pp}(\omega + k_x v) \hat{\mathbf{K}}_p^- \exp(ik_x x + ik_y y - iK_z z). \end{aligned} \quad (8)$$

Since we know the total wavefield we can calculate the force from the Maxwell stress tensor in vacuo,

$$T_{ij} = \frac{1}{2} \left\{ +\varepsilon_0 E_i E_j^* + \varepsilon_0 E_i^* E_j - \varepsilon_0 \delta_{ij} \mathbf{E} \cdot \mathbf{E}^* + \mu_0 H_i H_j^* + \mu_0 H_i^* H_j - \mu_0 \mathbf{H} \cdot \mathbf{H}^* \right\}. \quad (9)$$

The H -field being parallel to the surface makes no contribution to the force acting across the plane of the surface, but the electric fields give

$$\begin{aligned} F_x &= 2 |E_0|^2 \frac{\varepsilon_0 c_0^2}{\omega^2} k_x \sqrt{k_x^2 + k_y^2} \Im R_{1pp}(\omega + k_x v) \\ F_z &= 2 |E_0|^2 \frac{\varepsilon_0 c_0^2}{\omega^2} (k_x^2 + k_y^2) \Re R_{1pp}(\omega + k_x v). \end{aligned} \quad (10)$$

Note how the x -component of the force vanishes unless there is some dissipation in the system.

Next we ask what might be the source of the incident wavefield? If a second surface is brought close to the first surface there will be a wavefield outside this second surface whose amplitude is given at $T = 0$ in the following way. Let us normalize each mode in the system to have one unit of energy. Since the system is assumed translationally invariant parallel to the surface, each mode is characterized by a wave vector \mathbf{k} parallel to the surface. Let us suppose that normalized modes with frequencies between ω and $\omega + d\omega$ have an electric field density immediately outside the surface of

$$\frac{1}{\varepsilon_0} \frac{dN(\mathbf{k}, \omega)}{d\omega} d\omega \quad (11)$$

then, in a system in which the modes each contain the zero-point energy, the electric field density with frequencies between ω and $\omega + d\omega$ will be

$$|E_0|^2 = \frac{\hbar|\omega|}{2\varepsilon_0} \frac{dN(\mathbf{k}, \omega)}{d\omega} d\omega \quad (12)$$

where the prefactor of $\hbar|\omega|/2\varepsilon_0$ normalizes the wavefield to a total energy of $\hbar|\omega|/2$ per mode. Wijnands *et al* [25] show how to use Green's function theory to express the density of states at a distance d from the second surface in terms of the reflection coefficient of the surface,

$$\frac{dN(\mathbf{k}, \omega)}{d\omega} = \frac{\exp(-2kd)}{2\pi c_0^2 k} \omega \Im R_{2pp}(\omega) \quad (13)$$

where we have assumed that the second surface is stationary in our frame of reference. Hence on substituting

$$|E_0|^2 = \frac{\hbar|\omega|}{2\varepsilon_0} \frac{\exp(-2kd)}{2\pi c_0^2 k} \omega \Im R_{2pp}(\omega) d\omega \quad (14)$$

gathering together terms and integrating the forces over all frequencies and momenta parallel to the surface,

$$\begin{aligned} F_x &= 4 \sum_{k_x, k_y} \int_{-\infty}^{+\infty} \frac{\hbar|\omega|}{2\varepsilon_0} \frac{\exp(-2kd)}{2\pi c_0^2} \frac{\varepsilon_0 c_0^2}{\omega} k_x \Im R_{1pp}(\omega + k_x v) \Im R_{2pp}(\omega) d\omega \\ F_z &= 2 \sum_{k_x, k_y} \int_{-\infty}^{+\infty} \frac{\hbar|\omega|}{2\varepsilon_0} \frac{\exp(-2kd)}{2\pi c_0^2} \frac{\varepsilon_0 c_0^2}{\omega} k \Im [R_{1pp}(\omega + k_x v) R_{2pp}(\omega)] d\omega. \end{aligned} \quad (15)$$

We have also added a factor of two because of a symmetrical process in which zero-point waves emitted from surface one exert a force on surface two. Note the similarity between

these two forces. The first, representing the frictional force between the two surfaces, vanishes when $v = 0$ because of a symmetrical summation over an antisymmetrical function of k_x . The second represents the conventional Van der Waals force as, for example, derived in [26] and remains finite when $v = 0$.

Had we wished to study the effect of temperature we could have modified (12) to include the thermal contribution to radiation emitted from a free surface.

We can simplify the expression for the frictional force as follows. First we transform the summation over \mathbf{k} to an integral,

$$\begin{aligned}
 F_x &= \frac{\hbar}{\pi} \frac{L^2}{(2\pi)^2} \int_{-\infty}^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \\
 &\quad \times \int_{-\infty}^{+\infty} \Im R_{1pp}(\omega + k_x v) \Im R_{2pp}(\omega) \operatorname{sgn}(\omega) d\omega \\
 &= \frac{\hbar}{\pi} \frac{L^2}{(2\pi)^2} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \\
 &\quad \times \int_{-\infty}^{+\infty} [\Im R_{1pp}(\omega + k_x v) - \Im R_{1pp}(\omega - k_x v)] \Im R_{2pp}(\omega) \operatorname{sgn}(\omega) d\omega \quad (16)
 \end{aligned}$$

where we have exploited the symmetry of the reflection coefficient which follows from general principles of causality,

$$\Im R(-\omega) = -\Im R(+\omega). \quad (17)$$

Next we make a similar rearrangement of the frequency integration to give

$$\begin{aligned}
 F_x &= \frac{\hbar}{\pi} \frac{L^2}{(2\pi)^2} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \\
 &\quad \times 2 \int_0^{+\infty} [\Im R_{1pp}(\omega + k_x v) - \Im R_{1pp}(\omega - k_x v)] \Im R_{2pp}(\omega) d\omega. \quad (18)
 \end{aligned}$$

One final manipulation gives

$$\begin{aligned}
 F_x &= \frac{\hbar}{\pi} \frac{L^2}{(2\pi)^2} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \\
 &\quad \times 2 \left[\int_0^{+\infty} \Im R_{1pp}(\omega + k_x v) \Im R_{2pp}(\omega) d\omega \right. \\
 &\quad \left. - \int_0^{+\infty} \Im R_{1pp}(\omega - k_x v) \Im R_{2pp}(\omega) d\omega \right] \\
 &= \frac{\hbar}{\pi} \frac{L^2}{(2\pi)^2} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \\
 &\quad \times 2 \left[\int_{k_x v}^{+\infty} \Im R_{1pp}(\omega) \Im R_{2pp}(\omega - k_x v) d\omega \right. \\
 &\quad \left. - \int_0^{+\infty} \Im R_{1pp}(\omega - k_x v) \Im R_{2pp}(\omega) d\omega \right]. \quad (19)
 \end{aligned}$$

Obviously we could have made the same manipulations with the role of the two surfaces

inverted,

$$\begin{aligned}
 F_x &= \frac{\hbar}{\pi} \frac{L^2}{(2\pi)^2} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \\
 &\quad \times 2 \int_0^{+\infty} [\Im R_{1pp}(\omega + k_x v) - \Im R_{1pp}(\omega - k_x v)] \Im R_{2pp}(\omega) d\omega \\
 F_x &= \frac{\hbar}{\pi} \frac{L^2}{(2\pi)^2} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \\
 &\quad \times 2 \left[\int_{k_x v}^{+\infty} \Im R_{2pp}(\omega) \Im R_{1pp}(\omega - k_x v) d\omega \right. \\
 &\quad \left. - \int_0^{+\infty} \Im R_{2pp}(\omega - k_x v) \Im R_{1pp}(\omega) d\omega \right].
 \end{aligned} \tag{20}$$

Adding the two expressions gives

$$F_x = 4\hbar \frac{L^2}{(2\pi)^3} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \int_0^{k_x v} \Im R_{2pp}(\omega) \Im R_{1pp}(k_x v - \omega) d\omega. \tag{21}$$

We take the standard expression for the reflection coefficients of a dielectric surface, for example from Nordling and Österman [27],

$$\begin{aligned}
 R_{ss} &= + \frac{\cos(\theta_i) - \sqrt{\varepsilon} \cos(\theta_r)}{\cos(\theta_i) + \sqrt{\varepsilon} \cos(\theta_r)} \\
 R_{pp} &= - \frac{\cos(\theta_r) - \sqrt{\varepsilon} \cos(\theta_i)}{\cos(\theta_r) + \sqrt{\varepsilon} \cos(\theta_i)}
 \end{aligned} \tag{22}$$

where the angles are now complex,

$$\begin{aligned}
 \cos(\theta_i) &= \frac{K_z(\text{vac})}{|\mathbf{K}(\text{vac})|} = \frac{i\sqrt{k_x^2 + k_y^2 - \omega^2 c_0^{-2}}}{\omega c_0^{-1}} \approx \frac{i\sqrt{k_x^2 + k_y^2}}{\omega c_0^{-1}} \\
 \cos(\theta_r) &= \frac{K_z(\varepsilon)}{|\mathbf{K}(\varepsilon)|} = \frac{i\sqrt{k_x^2 + k_y^2 - \omega^2 \varepsilon c_0^{-2}}}{\omega \sqrt{\varepsilon} c_0^{-1}} \approx \frac{i\sqrt{k_x^2 + k_y^2}}{\omega \sqrt{\varepsilon} c_0^{-1}}.
 \end{aligned} \tag{23}$$

Hence,

$$\begin{aligned}
 \lim_{k_x^2 + k_y^2 \rightarrow \infty} R_{ss} &= 0 \\
 \lim_{k_x^2 + k_y^2 \rightarrow \infty} R_{pp} &= \frac{\varepsilon - 1}{\varepsilon + 1}
 \end{aligned} \tag{24}$$

and

$$\begin{aligned}
 F_x &= 4\hbar \frac{L^2}{(2\pi)^3} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y \\
 &\quad \times \int_0^{k_x v} \Im \frac{\varepsilon_1(\omega) - 1}{\varepsilon_1(\omega) + 1} \Im \frac{\varepsilon_2(k_x v - \omega) - 1}{\varepsilon_2(k_x v - \omega) + 1} d\omega
 \end{aligned} \tag{25}$$

which is the same as our more hard-won expression of section 4. Note, however, that the formulation in terms of reflection coefficients is more general as it makes no assumption whatever about the internal structure of the surfaces. This may be important when

considering systems in which the surface is intrinsically different from the bulk, surfaces with thin coatings being a pertinent example.

Within this simple formulation it is easy to correct for higher-order perturbations. We saw above how quantum fluctuations in a second surface result in a fluctuating wavefield incident on the first surface, with amplitude E_0 . However, we can identify further contributions to the incident amplitude from waves that are reflected from the first surface, and then from the second surface to return again. The process can be repeated to give a corrected incident amplitude,

$$\begin{aligned} E'_0 &= E_0 \left[1 + e^{-2kd} R_{2pp}(\omega) R_{1pp}(\omega + k_x v) + \{e^{-2kd} R_{2pp}(\omega) R_{1pp}(\omega + k_x v)\}^2 + \dots \right] \\ &= E_0 \left[1 - e^{-2kd} R_{2pp}(\omega) R_{1pp}(\omega + k_x v) \right]^{-1}. \end{aligned} \quad (26)$$

Hence correcting equation (15) for multiple scattering,

$$\begin{aligned} F_x &= 4 \sum_{k_x, k_y} \int_{-\infty}^{+\infty} \frac{\hbar|\omega| \exp(-2kd)}{2\varepsilon_0} \frac{\varepsilon_0 c_0^2}{2\pi c_0^2} \frac{\varepsilon_0 c_0^2}{\omega} k_x \Im R_{1pp}(\omega + k_x v) \Im R_{2pp}(\omega) \\ &\quad \times \left| 1 - e^{-2kd} R_{2pp}(\omega) R_{1pp}(\omega + k_x v) \right|^{-2} d\omega \\ F_z &= 2 \sum_{k_x, k_y} \int_{-\infty}^{+\infty} \frac{\hbar|\omega| \exp(-2kd)}{2\varepsilon_0} \frac{\varepsilon_0 c_0^2}{2\pi c_0^2} \frac{\varepsilon_0 c_0^2}{\omega} k \Im [R_{1pp}(\omega + k_x v) R_{2pp}(\omega)] \\ &\quad \times \left| 1 - e^{-2kd} R_{2pp}(\omega) R_{1pp}(\omega + k_x v) \right|^{-2} d\omega. \end{aligned} \quad (27)$$

In the examples that we shall consider the surfaces are mainly resistive, and multiple-scattering corrections make only a quantitative change to our predictions even when the surfaces are close together. However, when the surfaces support local modes, the frequencies of these modes can be drastically shifted by proximity of a second surface strongly affecting their contribution to friction.

3. A classical Hamiltonian description of moving surfaces

We have restricted ourselves to a system defined purely in terms of a classical macroscopic quantity: the dielectric constant, $\varepsilon(\omega)$. Therefore it is appropriate that we begin by constructing the classical equations of motion of the system, before following the well worn path of quantization via the Hamiltonian. We begin by constructing a Lagrangian, then define momentum coordinates which are used to find the Hamiltonian.

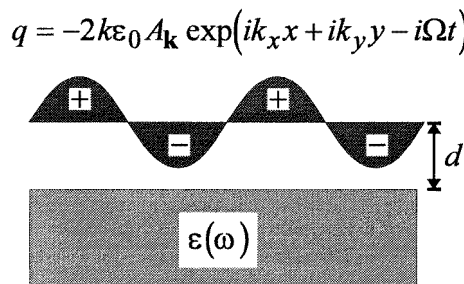


Figure 3. A sheet of charge sits a distance d above a dielectric surface.

First consider a classical system comprising a surface of a dielectric material in vacuo. We assume that the dielectric is a dynamic object defined by a continuum of harmonic

oscillator modes. These modes are vital to our subsequent calculations because they will be responsible for transporting energy away from the surface in the frictional process. Although we shall define the modes through the losses that they produce, that will also fix the real part of the dielectric function through causality as realized in the Kramers–Kronig relationships. Since we assume that the surface is translationally invariant, each mode is defined partly by a wave vector parallel to the surface, \mathbf{k} , and partly by a second subscript, j , which may be associated with degrees of freedom normal to the surface, and is responsible for transportation of energy away from the surface. We need say nothing about the nature of the modes other than how they couple to the outside world and this we probe with a sheet of charge placed a distance d above the surface (see figure 3). We can now write down the Lagrangian,

$$L_1 = T - V = \sum_{jk} \dot{s}_{jk}^2 - \omega_{jk}^2 s_{jk}^2 - A_k \beta_{jk} s_{jk} \exp(-kd - i\Omega t). \quad (28)$$

The first two terms on the right-hand side define the harmonic oscillators. Although they are written for simplicity as a discrete summation over modes, we shall always take the continuum limit. The last term on the right-hand side represents the coupling of each mode to the external charge. Note that it drops off exponentially with distance from the surface: at this stage we are mainly concerned with very-short-wavelength modes whose fields in the vacuum are largely electrostatic and therefore

$$\begin{aligned} k_x^2 + k_y^2 + k_z^2 &= 0 \\ k_z &= i\sqrt{k_x^2 + k_y^2}. \end{aligned} \quad (29)$$

The point of introducing the external charge is to probe the electrical activity of the modes in the vacuum. Since for any frictional process we are interested in the mutual excitation of modes across the intervening vacuum, all relevant coupling must pass through the vacuum and is therefore probed by our test charge. We can calculate the coupling parameter, β_{jk} , by calculating P , the rate of dissipation of energy in the dielectric, in two ways then equating the results.

First we use the Lagrangian equations of motion to calculate the dissipation from the rate of change of the external charge,

$$i\Omega 2k\epsilon_0 A_k \exp(-kd - i\Omega t) \quad (30)$$

multiplied by the voltage at the surface due to the j th mode defined to be

$$-\frac{1}{2k\epsilon_0} \beta_{jk} s_{jk}^* \exp(-kd). \quad (31)$$

Using the Lagrangian formalism to calculate the amplitude of the j th mode induced by the external charge,

$$s_{jk} = \frac{1}{2} \frac{\beta_{jk} A_k \exp(-i\Omega t)}{\Omega^2 - \omega_{jk}^2} \quad (32)$$

enables us to find

$$P_k = \frac{\Omega A_k^2}{2} \Im \sum_j \frac{\beta_{jk}^2}{\Omega^2 - \omega_{jk}^2}. \quad (33)$$

At first sight it may appear that the rate of power dissipation is always zero because the excitation frequency, Ω , will almost always differ from the mode frequencies, ω_{jk} , and hence the summation is real with no imaginary component. However, if we take the limit

of the poles very closely spaced together, and assume that the source of the excitation is switched on with a slowly increasing exponential, then we retrieve a finite answer on closing the contour in the lower half-plane:

$$P_k = \frac{\Omega A_k^2}{2} \Im \int_{-\infty}^{+\infty} \frac{\beta_{jk}^2}{(\Omega + i\delta)^2 - \omega_{jk}^2} \frac{dN_{jk}}{d\omega_{jk}} d\omega_{jk} = \frac{\pi A_k^2}{2} \beta_{jk}^2 \frac{dN_{jk}}{d\omega_{jk}}. \quad (34)$$

The physical interpretation of this result is that discrete modes are confined to the surface region and cannot transport energy away from the surface, but continuum modes reach into the bulk where they can dump energy.

Alternatively we may recognize that the test charge induces an image charge in the dielectric,

$$q' = + \left[\frac{\varepsilon(\Omega) - 1}{\varepsilon(\Omega) + 1} \right] 2k\varepsilon_0 A_k \exp(ik_x x + ik_y y - i\Omega t) \quad (35)$$

and therefore the loss can be found from the rate of working of the test charge,

$$P'_k = 2A_k^2 k \Omega \varepsilon_0 \Im \left[\frac{\varepsilon(\Omega) - 1}{\varepsilon(\Omega) + 1} \right]. \quad (36)$$

Equating P and P' gives

$$\beta_{jk}^2 \frac{dN_{jk}}{d\omega_{jk}} = \frac{4k\Omega\varepsilon_0}{\pi} \Im \left[\frac{\varepsilon(\Omega) - 1}{\varepsilon(\Omega) + 1} \right] \quad (37)$$

where

$$\frac{dN_k}{d\omega_k} \quad (38)$$

is the density of modes of wave vector k .

Thus we know how the modes in a surface interact with a sheet of charge outside the surface. This enables us to calculate how the modes in two parallel surfaces interact across the gap separating them. From (31) we know the voltage and hence the electric field produced in the gap by a given mode. By bringing the surfaces towards one another from infinity and integrating the force resulting from the overlapping fields, the interaction can be found,

$$\frac{\beta_{jk}\beta_{j'k}}{2k\varepsilon_0} \exp(-kd) s_{jk1} s_{j'k2}. \quad (39)$$

Hence we can write down the Lagrangian for two parallel surfaces separated by a distance d , for the moment assumed stationary with respect to one another:

$$L_{12} = \sum_{jk} \dot{s}_{jk1}^2 - \omega_{jk}^2 s_{jk1}^2 + \sum_{j'k'} \dot{s}_{j'k'2}^2 - \omega_{j'k'}^2 s_{j'k'2}^2 - \sum_{jj'} \frac{\beta_{jk}\beta_{j'k}}{2k\varepsilon_0} \exp(-kd) s_{jk1} s_{j'k2}. \quad (40)$$

We assume that the two surfaces are identical. Note that the test charge has been removed and that a new term represents coupling between modes on opposite surfaces. Since the coupling is mediated by an electrostatic field in the vacuum, our previous calculation of the coupling parameter is valid here also.

Finally the two surfaces are set in motion relative to one another as shown in figure 1. We shall assume that the relative velocity is small compared to the velocity of light. Choosing a frame of reference in which the surfaces have equal and opposite velocities, $\frac{1}{2}v$:

$$L_{12}(t) = \sum_{jk} \dot{s}_{jk1}^2 - \omega_{jk}^2 s_{jk1}^2 + \sum_{j'k'} \dot{s}_{j'k'2}^2 - \omega_{j'k'}^2 s_{j'k'2}^2 - \sum_{jj'} \frac{\beta_{jk}\beta_{j'k}}{2k\varepsilon_0} \exp(-kd) s_{jk1} s_{j'k2} \exp(-ik_x vt). \quad (41)$$

Only the coupling term changes: modes on opposite surfaces grate against one another in a washboard effect generating a frequency of $k_x v$. It is this finite frequency that will induce transitions in the system causing dissipation of energy. It is worth noting that the relevant frequencies range from zero up to a cut-off which is imposed either by the exponential decay of the coupling,

$$\omega_{\max 1} = v/d \quad (42)$$

or, if d is very small, by the shortest-wavelength fluctuations in the dielectric, usually of the order of 10^{-10} m. Thus the relevant frequencies are of the order of 10^{+10} Hz for a shear velocity of 1 m s^{-1} .

The final step in the classical treatment is to extract an Hamiltonian from the Lagrangian. First we define canonical momenta,

$$u = \frac{\partial L}{\partial \dot{s}} = 2\dot{s} \quad (43)$$

which gives

$$\begin{aligned} H_{12}(t) &= \sum_{jk} u_{jk1} \dot{s}_{jk1} + \sum_{j'k'} u_{j'k'2} \dot{s}_{j'k'2} - L_{12}(t) \\ &= \sum_{jk} \frac{1}{4} u_{jk1}^2 + \omega_{jk}^2 s_{jk1}^2 + \sum_{j'k'} \frac{1}{4} u_{j'k'2}^2 + \omega_{j'k'}^2 s_{j'k'2}^2 \\ &\quad + \sum_{jj'} \frac{\beta_{jk} \beta_{j'k}}{2k\epsilon_0} \exp(-kd) s_{jk1} s_{j'k2} \exp(-ik_x vt). \end{aligned} \quad (44)$$

4. A quantum description of moving surfaces

Now we introduce quantum mechanics into our classical picture using the conventional identification of

$$u \rightarrow -i\hbar \frac{\partial}{\partial s} \quad (45)$$

giving for the time-dependent Schrödinger equation

$$\begin{aligned} i \frac{\partial}{\partial t} \Psi(t) &= H_{12}(t) \Psi(t) \\ &= + \sum_{jk} \left\{ -\frac{1}{4} \frac{\partial^2}{\partial^2 s_{jk1}} + \omega_{jk}^2 s_{jk1}^2 \right\} \Psi(t) + \sum_{j'k'} \left\{ -\frac{1}{4} \frac{\partial^2}{\partial^2 s_{j'k'2}} + \omega_{j'k'}^2 s_{j'k'2}^2 \right\} \Psi(t) \\ &\quad + \sum_{jj'} \frac{\beta_{jk} \beta_{j'k}}{2k\epsilon_0} \exp(-kd) s_{jk1} s_{j'k2} \exp(-ik_x vt) \Psi(t). \end{aligned} \quad (46)$$

Some general observations are in order. The final term in the Schrödinger equation is capable of creating excitations in the system of energy $\hbar k v$ as can be made apparent by defining annihilation and creation operators,

$$a_{jk1}^{\pm} = \frac{1}{\sqrt{2}} \left[-i \frac{\partial}{\partial s_{jk1}} \pm i s_{jk1} \right] \quad a_{j'k2}^{\pm} = \frac{1}{\sqrt{2}} \left[-i \frac{\partial}{\partial s_{j'k2}} \pm i s_{j'k2} \right] \quad (47)$$

hence

$$s_{jk1} s_{j'k2} = -\frac{1}{2} \left[a_{jk1}^+ - a_{jk1}^- \right] \left[a_{j'k2}^+ - a_{j'k2}^- \right] \quad (48)$$

so the interaction term becomes

$$\begin{aligned}
& + \sum_{jj'} \frac{\beta_{jk}\beta_{j'k}}{2k\varepsilon_0} \exp(-kd) s_{jk1} s_{j'k2} \exp(-ik_x vt) \Psi(t) \\
& = -\frac{1}{2} \sum_{jj'} \frac{\beta_{jk}\beta_{j'k}}{2k\varepsilon_0} \exp(-kd) \left[a_{jk1}^+ - a_{jk1}^- \right] \left[a_{j'k2}^+ - a_{j'k2}^- \right] \\
& \quad \times \exp(-ik_x vt) \Psi(t). \tag{49}
\end{aligned}$$

A system originally in the ground state can only absorb energy that is shared between the two surfaces because an excitation is created on each surface. Frictional energy will be emitted from the interface in correlated pairs of excitations.

If we suppose that the system is in the ground state,

$$\Psi = \Psi_0 = \prod_{jk} \left| \frac{2\omega_{jk}}{\pi\hbar} \right|^{1/4} \exp(-\omega_{jk}\hbar^{-1}s_{jk1}^2) \prod_{j'k'} \left| \frac{2\omega_{j'k'}}{\pi\hbar} \right|^{1/4} \exp(-\omega_{j'k'}\hbar^{-1}s_{j'k'2}^2) \tag{50}$$

then we can find the frictional forces by calculating the rate of excitation of the system. We do this in the first instance to second-order perturbation theory to calculate the rate of excitation into the first excited state (one excitation per surface!):

$$\begin{aligned}
\Psi_{JKJ'K'} & = \left| \frac{32\omega_{JK}^3}{\pi\hbar^3} \right|^{1/4} s_{JK1} \exp(-\omega_{JK}\hbar^{-1}s_{JK1}^2) \prod_{jk \neq JK} \left| \frac{2\omega_{jk}}{\pi\hbar} \right|^{1/4} \exp(-\omega_{jk}\hbar^{-1}s_{jk1}^2) \\
& \quad \times \left| \frac{32\omega_{J'K'}^3}{\pi\hbar^3} \right|^{1/4} s_{J'K'2} \exp(-\omega_{J'K'}\hbar^{-1}s_{J'K'2}^2) \\
& \quad \times \prod_{j'k' \neq J'K'} \left| \frac{2\omega_{j'k'}}{\pi\hbar} \right|^{1/4} \exp(-\omega_{j'k'}\hbar^{-1}s_{j'k'2}^2). \tag{51}
\end{aligned}$$

The shift in the ground state energy can be written as

$$\Delta E = \sum_{k_x k_y} \frac{1}{4k^2\varepsilon_0^2} \exp(-2kd) \sum_{JJ'} \beta_{JK}^2 \beta_{J'K}^2 \left| \frac{1}{16\omega_{JK}\omega_{J'K}} \right| \frac{2|\omega_{JK} + \omega_{J'K}|}{|\omega_{JK} + \omega_{J'K}|^2 - k_x^2 v^2}. \tag{52}$$

The imaginary part of ΔE gives the rate of excitation out of the ground state; therefore we can calculate the rate of working:

$$\begin{aligned}
F_x v & = \frac{dU}{dT} = \sum_{k_x k_y} \frac{1}{2\pi^2} \exp(-2kd) \\
& \quad \times \int_{-\infty}^{+\infty} d\omega \operatorname{sgn}(\omega) \Im \left[\frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1} \right] \int_{-\infty}^{+\infty} d\omega' \operatorname{sgn}(\omega') \Im \left[\frac{\varepsilon(\omega') - 1}{\varepsilon(\omega') + 1} \right] \\
& \quad \times \Im \frac{2\hbar |\omega + \omega'|^2}{|\omega + \omega'|^2 - k_x^2 v^2 + i\eta} \tag{53}
\end{aligned}$$

where we have substituted for β_{JK}^2 the expression that we calculated earlier. Performing the ω' -integration:

$$F_x = \frac{\hbar}{\pi} \sum_{k_x k_y} \int_{-\infty}^{+\infty} \exp(-2kd) k_x \Im \left[\frac{\varepsilon(k_x v - \omega) - 1}{\varepsilon(k_x v - \omega) + 1} \right] \Im \left[\frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1} \right] d\omega. \tag{54}$$

We recognize in equation (54) the expression for the frictional force derived in equation (15) of section 2.

5. The nature of quantum friction

First some general observations about our formula for friction. We note that there must be loss processes operating in both surfaces. Note also the similarity of our formula for friction to the expression derived earlier for the Van der Waals force. However, the significant difference in the frictional force is that the imaginary part of the response is used.

We can make some statements about the interrelationship of distance and velocity dependence of friction. Note that on making the substitution

$$\mathbf{k}' = \mathbf{k}v \quad (55)$$

our formula (21) can be rewritten:

$$F_x = 4\hbar \frac{L^2}{(2\pi)^3} \frac{1}{d^3} \frac{1}{v^3} \int_0^{+\infty} k'_x dk'_x \int_{-\infty}^{+\infty} \exp(-2k'dv^{-1}) dk'_y \\ \times \int_0^{k'_x} \Im R_{2pp}(\omega) \Im R_{1pp}(k'_x - \omega) d\omega \quad (56)$$

and hence

$$F_x = \frac{1}{d^3} g\left(\frac{v}{d}\right) \quad (57)$$

where g is an arbitrary function. If we assume that the particular form of the dielectric function results in a power-law dependence on the velocity it follows that

$$F \propto \frac{v^\mu}{d^{\mu+3}}. \quad (58)$$

Therefore the v -dependence and d -dependence are interlinked. This may be a way of identifying these contributions to friction.

Now let us explore what happens with various standard forms of the dielectric function.

5.1. Constant $\varepsilon(\omega)$

This is the simplest case, but possibly the least physical, and results in a frictional force of the form

$$F = \left[\Im \frac{\varepsilon - 1}{\varepsilon + 1} \right]^2 \frac{3\hbar v}{2^6 \pi^2 d^4}. \quad (59)$$

Note that our ‘rule of powers’ is obeyed and that friction has the form that would be expected if the vacuum behaved as a viscous fluid. The force falls rapidly as d^4 . Nevertheless there is no exponential decay of the force because interactions between the two surfaces are conveyed by a massless particle, the photon. Note also the central role played by

$$\Im \frac{\varepsilon - 1}{\varepsilon + 1} \quad (60)$$

which represents the reflection coefficient of the surface for p-polarized radiation, and is also proportional to the density of electromagnetic states immediately outside a surface.

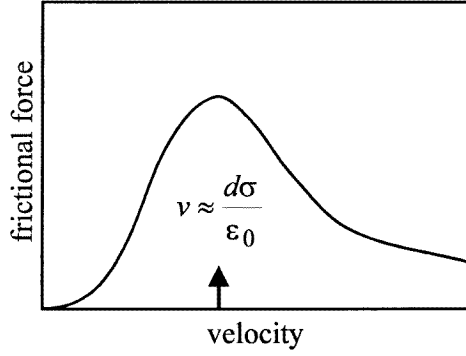


Figure 4. The frictional force between two conducting surfaces separated by distance d . At low velocities the force increases as v^3 , and at high velocities it decreases as $v^{-1} \ln v$, reaching a maximum at $v \approx d\sigma\epsilon_0^{-1}$.

5.2. Constant conductivity

In this case the dielectric function has the form

$$\epsilon = 1 + i \frac{\sigma}{\omega\epsilon_0} \quad (61)$$

where σ is the conductivity. For this case the integrals are tricky to evaluate except in the limiting cases of high and low velocities. However, we sketch the qualitative form of the force in figure 4, and quote the limiting cases:

$$F = \frac{5\hbar\epsilon_0^2 v^3}{2^8 \pi^2 \sigma^2 d^6} \quad v \ll \frac{d\sigma}{\epsilon_0} \quad (62)$$

$$F = \frac{\hbar\sigma^2}{32d^2\pi^2 v\epsilon_0^2} \ln\left(\frac{v\epsilon_0}{2d\sigma}\right) \quad v \gg \frac{d\sigma}{\epsilon_0}.$$

Since this is a more realistic instance, it is worth evaluating the force. We pause a moment to maximize the effect which we achieve approximately by maximizing

$$\Im \frac{\epsilon - 1}{\epsilon + 1} = \Im \frac{i\sigma/\omega\epsilon_0}{2 + i\sigma/\omega\epsilon_0} = \frac{2\sigma/\omega\epsilon_0}{4 + (\sigma/\omega\epsilon_0)^2} \quad (63)$$

i.e. we choose

$$\sigma = 2\omega\epsilon_0 \approx 2kv\epsilon_0 \approx 2d^{-1}v\epsilon_0 \quad (64)$$

where we have recognized that the critical frequencies will be the highest for which loss occurs. Assuming a modest shear velocity and surfaces in atomic contact gives

$$d = 10^{-10} \text{ m} \quad (65)$$

$$v = 1.0 \text{ m s}^{-1}$$

where we have recognized that the dielectric response of the surface will cut off at something like a screening length, typically 10^{-10} m in a metal. Under these conditions we choose

$$\sigma = 0.1 \Omega^{-1} \text{ m}^{-1} \quad (66)$$

which puts us more or less at the crossover point of the two limiting formulae, and at the maximum of the friction/velocity curve in figure 4. This sort of conductivity is not

untypical of semi-metals such as carbon. We make a rough estimate of the frictional force by substituting into the high-velocity formula (the least sensitive to the velocity) and find

$$F \approx 3 \times 10^3 \text{ N m}^{-2}. \tag{67}$$

However, this result needs to be qualified: no two surfaces placed in contact will actually touch over their entire area. Estimates of the fractional area in contact vary around 0.001. Therefore if we assume that our surfaces have only this fractional intimate contact the observed force will be much smaller,

$$F_{\text{obs}} \approx 3 \text{ N m}^{-2}. \tag{68}$$

In other words for a restricted class of materials, the semi-metals, in which the electromagnetic density of states outside the surface is maximized in the relevant frequency range, this contribution to frictional forces is substantial. Furthermore, because of the power-law dependence on the separation of surfaces, it will dominate the long-range contributions as the other contribution, due to exchange of electrons, must always have an exponential decay in consequence of the finite work function of the electron.

5.3. Frictional forces independent of velocity

The dielectric functions discussed above all lead to frictional forces *dependent* on the velocity, but experiment mainly measures forces *independent* of velocity. It is interesting to speculate on whether our model of photon exchange can reproduce velocity independence and under what circumstances.

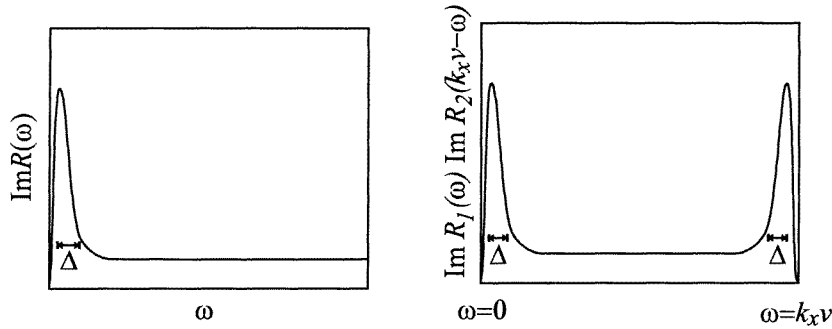


Figure 5. The reflection coefficient shown on the left implies that the surface can sustain many low-frequency excitations which absorb momentum without absorbing much energy. At finite frequencies the coefficient is nearly constant. On the right is shown the corresponding frequency integrand in equation (20): if the peaks are strong enough the integral will be independent of v .

Inspecting equation (21) for the force we can see that the velocity dependence would be eliminated if the reflection coefficients had the form shown in figure 5. Under these circumstances the frequency integrand in (21) is sketched in figure 5. If we assume that the peaks dominate the integration, then the result will be independent of velocity. Of course this assumes a finite velocity such that the peaks are well separated.

We might further speculate on the nature of a surface with a capacity to absorb low-frequency radiation with large components of momentum parallel to the surface. Surface scientists are fond of invoking a ‘dirty’ surface and such an object exactly meets our requirements in this case. A surface on which there is a random assortment of loose massive fragments will absorb a great deal of momentum for little energy input. The

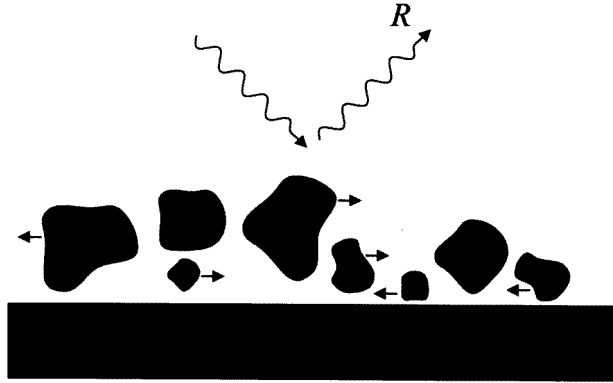


Figure 6. A model of a 'dirty' surface: low-frequency excitations of massive particles on the surface absorb plenty of momentum but little energy leading to a low-frequency peak in absorption of incident radiation.

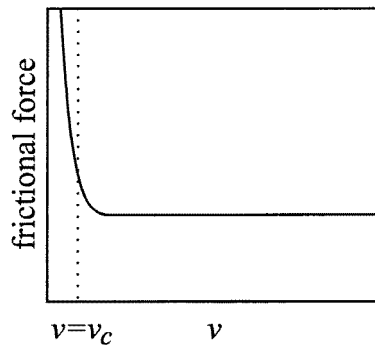


Figure 7. The frictional force as a function of velocity as predicted by our model of a 'dirty' surface.

fragments need only be massive relative to the fundamental particles, and nanometre-sized lumps would be perfectly adequate. Figure 6 shows our model of a dirty surface, and we suggest that the electromagnetic reflection coefficient would show the low-frequency peak sketched in figure 5. The width of the low-frequency peak could then be predicted in terms of the mass, M , of the fragments:

$$\Delta \approx \frac{\hbar k_{\max}^2}{M}. \quad (69)$$

Interestingly enough, at low velocities the model predicts that the frictional force will rise rapidly when the peaks overlap. This will happen when

$$k_x v \approx \Delta \quad (70)$$

where Δ is the width of the low-frequency peak. The effect is shown in figure 7. The critical velocity at which sliding friction changes into static friction is predicted by the width, Δ , of the low-frequency peak in reflectivity:

$$k_{\max} v_c \approx \Delta \quad (71)$$

where k_{\max} is the largest wave vector that can be excited in the system, which we expect to be of the order of 10^{+11} m^{-1} .

6. Other contributions to friction

Our perspective is that friction arises through exchange of momentum-carrying particles between surfaces. We have considered the photon, but other exchanges are possible: atoms may be exchanged as happens in the case of friction with wear [28, 29], but the most obvious competitor to the photon is the electron. Note that the *phonon*, as opposed to the *photon*, is not a particle that can have any existence separate from the solid; it does not exist in vacuum and therefore is not on the list of exchangeable particles, though phonon exchange can be mediated by a another particle.

The interaction of two surfaces through exchange of electrons is a vast subject and covers nearly the whole of chemical bonding at surfaces. Therefore, since we are only interested in a simple comparison with the photon case, we choose the most elementary possible model capable of generating friction within the context of two smooth translationally invariant surfaces. We shall assume the surfaces to be made of the same material, and the electrons to be defined by a spherical Fermi surface.

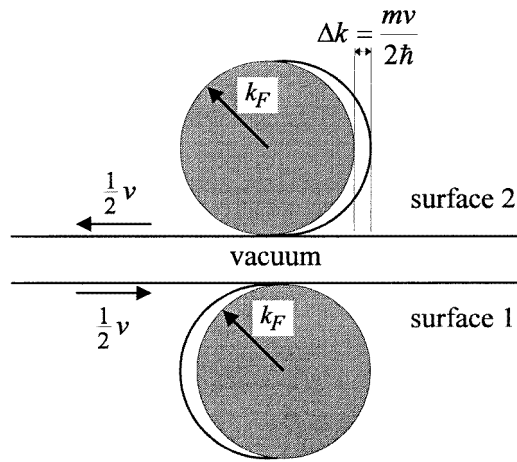


Figure 8. When two surfaces move relative to one another, the Fermi surfaces shift slightly in the direction of movement, creating a thin annulus of states into which electrons can tunnel from the other surface. This creates a momentum exchange and hence a force between the surfaces.

When the two surfaces are at rest relative to one another, there is no exchange of electrons because the exclusion principle forbids tunnelling into filled states. When the surfaces are in relative motion a small slit of states appears on either side of the Fermi surfaces, where tunnelling is allowed; see figure 8. We can work out the rate at which the surfaces exchange momentum through this tunnelling mechanism:

$$F_x = 2 \int_0^\pi \hbar k_F^4 v \frac{L^2}{(2\pi)^3} \cos^2(\theta) \sin^2(\theta) \exp \left[-2d \sqrt{\frac{2m\phi}{\hbar^2} + k_F^2 \cos^2 \theta} \right] d\theta \quad (72)$$

where d is the separation between the two surfaces, ϕ is the work function relative to vacuum, k_F is the Fermi momentum, m is the electronic mass, v is the shear velocity, and θ is the angle between the electron momentum and v .

In the limit in which the surfaces are in contact,

$$F_x = 2 \int_0^\pi \hbar k_F^4 v \frac{L^2}{(2\pi)^3} \cos^2(\theta) \sin^2(\theta) d\theta = \frac{1}{2} \int_0^\pi \hbar k_F^4 v \frac{L^2}{(2\pi)^3} \sin^2(2\theta) d\theta$$

$$= \frac{\pi}{4} \hbar k_F^4 v \frac{L^2}{(2\pi)^3} = \frac{L^2}{32\pi^2} \hbar k_F^4 v \quad (73)$$

Substituting a Fermi momentum typical of aluminium and a typical shear velocity,

$$\begin{aligned} k_F &= 1 \text{ au} \approx 10^{+10} \text{ m}^{-1} \\ v &= 1 \text{ m s}^{-1} \end{aligned} \quad (74)$$

the force is

$$L^{-2} F_x = \frac{1}{32\pi^2} \hbar k_F^4 v = \frac{10^{-34} \times 10^{+40} \times 1}{32\pi^2} = 3.16 \times 10^{+3} \text{ N m}^{-2}. \quad (75)$$

Note that this electronic contribution to friction is comparable in magnitude to that obtained from the tunnelling of photons: compare equation (67).

The main difference between photonic and electronic friction is the dependence on separation, d , between the surfaces. In the photonic case there is nearly always a power law whereas in the electronic case the existence of a finite work function dictates that the force decays exponentially with d . Photons always dominate at large distances.

7. Can sheared interfaces emit light?

The friction that we have discussed so far involves frequencies typically in the GHz range, much lower than optical frequencies. The frequency is limited by k , the wave vector parallel to the surface of the radiation, and the relative velocity of the surfaces,

$$\omega \leq kv \quad (76)$$

as we have already discussed. Since the screening length in a material is of the order of the separation between electrons, the highest frequencies possible at shear velocities of the order of 1 m s^{-1} are no more than a few GHz. Therefore there is no emission of visible light by this mechanism unless the shear velocity is impossibly large.

GHz microwaves can be produced, but to be observed must be ejected into a transparent dielectric and have a real wave vector:

$$K'_z = \sqrt{\varepsilon\omega^2 c_0^{-2} - k_x^2 - k_y^2 + i\delta} \quad (77)$$

where the prime denotes that we are in the dielectric. For K'_z to be real,

$$\varepsilon\omega^2 c_0^{-2} \approx \varepsilon k^2 v^2 c_0^{-2} > k_x^2 + k_y^2 \quad (78)$$

which requires

$$\varepsilon > \left(\frac{c_0}{v}\right)^2. \quad (79)$$

Our conclusion is that, unless we assume improbably large velocities or huge values of ε , no free radiation can be emitted from smooth sheared surfaces. The situation is different if we allow the surfaces to be rough, but that is beyond the scope of this paper.

8. Conclusions

We have shown that friction exists between sheared smooth dielectric surfaces at $T = 0$, provided that we take quantum fluctuations into account. If the surfaces are almost in physical contact, the frictional forces may be comparable to other contributions observed in everyday situations, provided that the surfaces have a high density of electromagnetic

states in the GHz region: in other words resistivity of the order of $1 \text{ m } \Omega$. Details of how friction depends on surface separation and velocity vary with the materials, but in general friction decays with a power-law dependence on surface separation, in a manner linked to the velocity dependence and can be expected to be the dominant frictional force at large separations, just as the Van der Waals force dominates in this regime.

Although finite temperatures are beyond the scope of this paper, we can expect some modifications to our conclusions at room temperature because the excitations created by friction are of the order of $k_B T_{\text{room}}$.

The formula for quantum friction (21) is very simple and given a few assumptions can be derived in a few lines of algebra alongside the well known formula for the attractive force between surfaces. A more careful quantum treatment associates quantum friction with emission of pairs of correlated photons, one into each surface, but gives the same formula as the rough approach.

Brief consideration was given to friction arising from exchange of electrons: a different mechanism from the electron–hole pair creation considered previously (the latter is in reality a photon-exchange process). Tunnelling of electrons between surfaces can also give a substantial frictional force, but exponential decay of the electron wave functions means that these forces are shorter ranged than photon-based forces.

Finally we asked whether free light could be emitted from a sheared interface. It can but under extreme conditions of shear velocity, or of material properties that render it practically impossible. This conclusion is reached within the context of smooth surfaces. If the surfaces are rough on an atomic scale, the momentum conservation arguments lying behind our conclusions are no longer valid. Rough surfaces may emit light on being sheared.

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References

- [1] Bowden F P and Tabor D 1967 *Friction and Lubrication* (London: Methuen)
- [2] Persson B N J and Tosatti E (ed) 1995 *Physics of Sliding Friction (NATO ASI Series)* (Dordrecht: Kluwer)
- [3] Ritchie R H and Howie A 1988 *Phil. Mag. A* **58** 753
- [4] Ferrell T L, Echenique P M and Ritchie R H 1979 *Solid State Commun.* **32** 419
- [5] Echenique P M and Pendry J B 1975 *J. Phys. C: Solid State Phys.* **8** 2936
- [6] Schaich W L 1974 *Solid State Commun.* **15** p 357
- [7] Guinea F 1984 *Phys. Rev. Lett.* **53** p 1268
- [8] Pendry J B and Martín-Moreno L 1994 *Phys. Rev. B* **50** 5062
- [9] Sols F, Miranzo P and Flores F 1985 *Surf. Sci.* **161** 33
- [10] Barton G and Eberlein C 1993 *Ann. Phys., NY* **227** 222
- [11] Schwinger J 1992 *Proc. Natl Acad. Sci. USA* **89** 4091
- [12] Eberlein C 1996 *Phys. Rev. Lett.* **76** 3842
- [13] Weninger K R, Barber B P and Putterman S J 1997 *Phys. Rev. Lett.* **78** 1799
- [14] Schaich W L and Harris J 1981 *J. Phys. F: Met. Phys.* **11** 65
- [15] Teodorovitch E V 1978 *Proc. R. Soc. A* **362** 71
- [16] Levitov L S 1989 *Europhys. Lett.* **8** 499
- [17] Annett J F and Echenique P M 1986 *Phys. Rev. B* **34** 6853
- [18] Annett J F and Echenique P M 1986 *Phys. Rev. B* **36** 8986
- [19] Barton G 1996 *Ann. Phys., NY* **245** 361

- [20] Polevoi V G 1990 *Sov. Phys.-JETP* **71** 1119
- [21] Mkrtchian V E 1995 *Phys. Lett.* **207** 299
- [22] Liebsch A 1997 *Phys. Rev. B* **55** 13 263
- [23] Høye J S and Brevik I 1992 *Physica A* **181** 413
- [24] Høye J S and Brevik I 1993 *Physica A* **196** 241
- [25] Wijnands F, Pendry J B, Roberts P J, Garcia-Vidal F J, Martín-Moreno L and Bell P M 1997 *Opt. Quantum Electron.* **29** 199
- [26] Abrikosov A A, Gor'kov L P and Dzyaloshinskii I Ye 1965 *Quantum Field Theoretical Methods in Statistical Physics* translated from the Russian by D E Brown and D ter Haar (Oxford: Pergamon)
- [27] Nordling C and Österman J 1980 *Physics Handbook* (Lund: Studentlitteratur)
- [28] Stoneham A M, Ramos M M D and Sutton A P 1993 *Phil. Mag. A* **67** 797
- [29] Stoneham A M and Harker A H 1982 *Wear* **80** 377